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tegers had assembled. One of them informed *XYZ* that their members had been added together and had multiplied rapidly and in some instances one had been subtracted from another or a family had been divided, but that in no case had a number been produced which did not belong to the set. "However, we are a discreet set," he added with a smile. "We are careful not to let the root-extractor enter our midst." The members then began discussing their figures, the figures by which they were recognized. "I maintain that I have the most peculiar shape," said 6, "See! when I stand on my head I become 9." "That 's nothing, said 8, "When I lie down I become infinite." "But then you are no longer one of us," replied the others, "What shall we do with him for perpetrating such a joke? We 'll leave it to you, Mr. *XYZ*." "Well," said *XYZ*, "I think he should be killed. Let him that is greatest among you cast the first stone." They were all silent. Then Zero rolled along and tried to start an argument. "You do n't amount to anything anyhow," the others protested. "Just the same," retorted Zero, "I claim a distinction of which none of you can boast: a distinguished human, Mr. Russell, has dedicated a chapter to me."

At this stage Mr. *XYZ*, regarding his mission as hopeless, moved away into space, more firmly grounded in his conviction that the Continuum will never be well-ordered.

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AN EXAMPLE OF THE USEFULNESS OF FOURIER'S THEOREM IN SEPARATING THE ROOTS OF EQUATIONS.

By L. R. MANLOVE, England.

Cases must occur in practice where the roots of an equation cannot be separated by any of the well-known easier methods and where Sturm's Theorem is inapplicable by reason of the labor which it involves. In such cases resort may be had to a combination of Fourier's Theorem with Lagrange's method of approximation, as shown below.

EXAMPLE.

Let us take the equation

$$x^{17}-35x^{15}+11x^{14}-1000x^{10}+2500x^{5}-151x^{3}+1=0.$$

A first application of Fourier's Theorem shows that there are:

(a) *Two positive roots:* One between 1 and 2; one between 5 and 6.

(b) *Three negative roots:* One between 0 and —1; one between —1 and —2; one between —6 and —7.

(c) *A doubtful interval* between 0 and $+1$ in which 4 changes of sign are lost and which *may* consequently include 4 more positive roots.

To dispose of the doubtful interval, put $x=\frac{1}{u_1}$.

The equation in u_1 is:

$$u_1^{17}-151u_1^{14}+2500u_1^{12}-1000u_1^{7}+11u_1^{3}-35u_1^{2}+1=0=\text{(say)}\,F_A(u_1).$$

Fourier's functions of $F_A(u_1)$ give for the value 1 of u_1 two changes of sign, and for the value 2 no changes of sign.

$F_A(u_1)=0$ may therefore have two roots between 1 and 2, but has no other positive roots greater than unity.

The four originally doubtful roots are reduced to two.

To dispose of the remaining pair put $u_1=1+\frac{1}{u_2}$.

The equation in u_2 is:

$$1327u_2^{17}+20866u_2^{16}+\text{(9 positive terms)}-12588u_2^{6}-5053u_2^{5}$$
$$+\text{(5 positive terms)}=0=\text{(say)}\ F_B(u_2),$$

which clearly can have no positive root greater than unity.

The four originally doubtful roots are all imaginary and the proposed equation is disposed of.

REMARKS.

The formation (by Horner's method) of the equation $F_B(u_2)=0$ is effected *currente calamo* and the whole example might probably be worked in less than an hour.

It will be observed that in the auxiliary equations used in this method we are only concerned with positive values of the variable greater than unity.

Negative roots of the original equation are most conveniently dealt with by substituting $-y$ for x in that equation and seeking the positive roots of the resulting equation in y.

The writer has tested the method by a number of trials and finds that for most intervals the doubtful roots are at or before the second auxiliary equation, either separated or shown to be imaginary, and in no case, so far, has he found it necessary to go beyond the third auxiliary equation.

Failure to attain definite results within the limits just specified creates a presumption of the existence of equal roots which must be dealt with in the ordinary way before proceeding farther.